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Electromagnetic splittings for hadrons with dressed constituent quarks

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Abstract. Electromagnetic splittings for hadrons are calculated in a formalism where the constituent quarks are considered as dressed quasiparticles. The electromagnetic interaction, which contains coulomb, contact and hyperfine terms, is folded with the quark electrical density. The strong potential is a modification of the well known funnel potential. Our model contains only one free parameter and the agreement with experimental data is reasonable although it seems very difficult to obtain a perfect description in any case.

PACS. 12.39Pn Potential models – 13.40Dk Electromagnetic process and properties

1 Introduction

Quantum Chromodynamics (QCD) is believed to be the good theory of strong interaction, but mesons and baryons belong to the non perturbative regime, and, in this case, the theory is very complicated. This explains why a number of alternative simpler models were invented.

Among them, the non relativistic quark model (NRQM) is very appealing: it is simple concerning both the formulation and the numerical calculation, it allows a good treatment of the center of mass motion and it has met with a lot of successes in many domains (see [1]).

In those models, the degrees of freedom are called constituent quarks; they are complicated objects and they must be considered as quasi-particles with some spatial extension.In this picture, the constituent quarks must have some spatial density and the strong interaction should be folded with some gluonic density and the electromagnetic potential with some electromagnetic quark density ([2],[3],[6],[7]). There is no reason that the strong density and the electromagnetic density should be the same.

One knows that it is very difficult to obtain a good description for the spectra of mesons and baryons in a unified treatment([8], [9]). In general, a model good for mesons fails for the description of baryons and the other way round. Here we need to have a correct description of both and we use a strong potential that is a good compromise for that.

But spectra are not enough to test completely a model, and one must rely on more sensitive observables ([10]). The electromagnetic splittings of isospin multiplets are very well suited to make such a study. The origin of the splitting is a mass difference between the up and down quark (probably already present in the original QCD lagrangian) and also an electromagnetic potential containing a coulomb term and relativistic corrections to it.

In this paper, we want to deal with all the known splittings both in mesons and in baryons in a consistent approach and to push the NRQM study further in several domains. First we want to perform a precise and complete treatment, avoiding perturbative expressions. Second we introduce the contact term, that is usually neglected, in the electromagnetic potential. Lastly, our most important improvement is the use of a dressed electromagnetic interaction among the quarks.

The first section is devoted to the description of strong and electromagnetic potentials that will be employed. The second section presents the results for the splittings of mesons and baryons and the last section deals with conclusions.

2 Strong and electromagnetic potentials

2.1 AL1 potential

Since we are interested by meson and baryon splittings, we have the necessity to use a strong potential that allows a good description of spectra for both types of hadrons. The AL1 potential ([11],[12]) is a slight modification of the well known funnel potential. It looks like :

$$V_{ij}(r) = -\frac{3}{16}\lambda_i \cdot \lambda_j \left[V_C^{(ij)}(r) + V_H^{(ij)}(r)\right]. \qquad (1)$$

Basically it contains two terms with the same colour dependence, that comes in fact from one gluon exchange.

The central part

$$V_C^{(ij)}(r) = -\frac{\kappa}{r} + ar + C. \tag{2}$$

is coulomb+linear and it has also a constant contribution, necessary to reproduce the absolute masses. There is no reason, except simplicity, that the colour dependence is kept unchanged for the confining and constant parts.

The hyperfine term

$$V_H^{(ij)}(r) = \frac{8\pi}{3m_i m_j}\kappa' \frac{\exp(-r^2/\sigma_{ij}^2)}{\pi^{3/2}\,\sigma_{ij}^3}\mathbf{s}_i \cdot \mathbf{s}_j. \tag{3}$$

is flavour dependent through the interacting masses; beside the usual $1/m_i m_j$ factor, the range σ_{ij} is also mass dependent: σ_{ij} does depend on the flavor

$$\sigma_{ij} = A\left(\frac{2m_i m_j}{m_i + m_j}\right)^{-B}. \tag{4}$$

The parameters are completely phenomenological and have been determined essentially on meson spectra. In this sense, they already contain the dressing by the gluonic density. This is manifest from the expression of the hyperfine term, in which the usual $\delta(\mathbf{r})$ term is regularized by a gaussian function.

This potential, used in a Schroedinger equation, gives an overall good description of hadronic spectra, both in the mesonic and baryonic sectors.

There exist many algorithms to compute radial wave functions. Due to the extreme sensitivity of the splitting on the numerical treatment, it is very important to adopt a method which is very precise (and if possible a fast one). In the mesonic sector, we used a method based on Lagrange mesh, which is very simple, very precise and very fast. Technical details of this method can be found in [4]. The number of significant digits is around 10. In the baryonic sector, we used a variational method based on harmonic oscillator basis with different sizes for the various Jacobi coordinates. This method was described in [5] and was proved to be competitive with the stochastic method, if pushed to a number of quanta in the basis equal to 20. The number of significant digits is estimated to 5.

2.2 Electromagnetic quark density

In NRQM, the constituent quarks are extended objects. This means that for a quark at an average position r, there exists a certain probability to be at position r': this probability is more or less a density $\rho(r - r', \gamma)$, where the parameter γ represents the size of the object. This density must be a peaked function reducing to a delta function at the limit of a vanishing size. Another natural property is that the density is isotropic. Lastly, we also require that its integral over the whole space is unity. The most popular densities are of lorentzian, gaussian or Yukawa type. Here we adopt a Yukawa form. There is a precise reason for that: the density is the leading ingredient of the meson charge form factor. It is an experimental fact that the data accomodate rather nicely a Yukawa density, which has the good asymptotic behaviour. The chosen form is such as :

$$\rho_i(\mathbf{u}) = \frac{1}{4\pi\gamma_i^2}\frac{e^{-u/\gamma_i}}{u}, \tag{5}$$

Within this framework, the two body potentials are obtained from the bare ones with help of a double convolution, one convolution for each of the interacting particle. In fact, just by a simple change of variables, such a double convolution can be reduced to a single convolution:

$$U_{ij}(\mathbf{r}) = \int d\mathbf{r}'\, U_{ij}^{(b)}(\mathbf{r}')\,\rho_{ij}(\mathbf{r} - \mathbf{r}'), \tag{6}$$

In the pecular case of our Yukawa density, the expression for the interacting density is :

$$\rho_{ij}(\mathbf{u}) = \frac{1}{4\pi(\gamma_i^2 - \gamma_j^2)}\left(\frac{e^{-u/\gamma_i}}{u} - \frac{e^{-u/\gamma_j}}{u}\right) \tag{7}$$

This expression is valid for interacting particles with different sizes. For particles with identical sizes, the true expression is just the limit of the previous one for $\gamma_j \to \gamma_i$.

2.3 Bare potential

To describe the splittings, one needs first a weak $SU(2)$ breaking allowing a different mass for the u and d quark, but also the presence of the electromagnetic potential. Its traditionnal form originates from relativistic corrections to the Coulomb potential.

$$U_{ij}^{(b)}(\mathbf{r}) = (U_{\text{coul}})_{ij}^{(b)}(\mathbf{r}) + (U_{\text{cont}})_{ij}^{(b)}(\mathbf{r}) + (U_{\text{hyp}})_{ij}^{(b)}(\mathbf{r}), \tag{8}$$

The important terms are the Coulomb, hyperfine and contact terms whose expressions are :

$$(U_{\text{coul}})_{ij}^{(b)}(\mathbf{r}) = Q_i Q_j \frac{\alpha}{r}, \tag{9}$$

$$(U_{\text{cont}})_{ij}^{(b)}(\mathbf{r}) = -\frac{\pi}{2} Q_i Q_j \left(\frac{1}{m_i^2} + \frac{1}{m_j^2}\right)\alpha\,\delta(\mathbf{r}), \tag{10}$$

$$(U_{\text{hyp}})_{ij}^{(b)}(\mathbf{r}) = -\frac{8\pi Q_i Q_j}{3m_i m_j}\alpha\,\delta(\mathbf{r})\mathbf{s}_i \cdot \mathbf{s}_j, \tag{11}$$

where α is the fine structure constant.

In principle it contains also Darwin, spin-orbit and tensor contributions, but their effects are presumably weak and they are neglected in the following. Usually people also neglect the contact term, keeping only the Coulomb and dipole-dipole interactions. Here we include the effect of the contact term in order to grasp quantitatively its effect and to justify a posteriori the validity (or not) of neglecting it.

2.4 Dressed potential

From the bare potential of the previous section 8, and the interacting density 7, the convolution 6 gives the expression of the dressed electromagnetic potential. For each contribution, the various terms look like this :

$$(U_{\text{coul}})_{ij}(r) = \alpha Q_i Q_j \left(\frac{1}{r} - \frac{\gamma_i^2}{\gamma_i^2 - \gamma_j^2} \frac{e^{-r/\gamma_i}}{r} + \frac{\gamma_j^2}{\gamma_i^2 - \gamma_j^2} \frac{e^{-r/\gamma_j}}{r} \right), \quad (12)$$

$$(U_{\text{cont}})_{ij}(r) = -\frac{\alpha Q_i Q_j}{8(\gamma_i^2 - \gamma_j^2)} \left(\frac{1}{m_i^2} + \frac{1}{m_j^2} \right) \frac{e^{-r/\gamma_i} - e^{-r/\gamma_j}}{r}, \quad (13)$$

$$(U_{\text{hyp}})_{ij}(r) = -\frac{2\alpha Q_i Q_j}{3 m_i m_j (\gamma_i^2 - \gamma_j^2)} \frac{e^{-r/\gamma_i} - e^{-r/\gamma_j}}{r} \mathbf{s}_i \cdot \mathbf{s}_j. \quad (14)$$

and the corresponding limits in the case of particles with identical sizes.

Very often such a potential is treated perturbatively. Here we have the ability to perform an exact treatment. A comparison with a perturbative calculation is very instructive.

3 Results

3.1 Determination of the parameters

The first thing to do is to determine the parameters. We do not want to introduce a lot of new parameters; here we restrict the number of free parameters to the minimum unavoidable. In particular, the parameters of the strong potential are maintained without modification. The first parameters to be introduced are the electromagnetic sizes of the quarks γ_i. Indeed the meson form form factors, or alternatively the charge mean square radii, are very dependent from those parameters. The dressed radii are just the sum of the bare radii plus a contribution entirely due to the dressing, which can be expressed, with the Yukawa density, as:

$$\langle r^2 \rangle = \langle r^2 \rangle^{(b)} + 6 \sum_{i=1}^{2} e_i \gamma_i^2. \quad (15)$$

Thus the pion radius is well suited to get the size for the u and d quarks; I suppose that these two sizes are identical. The kaon radius depends both on the size of the ordinary quark and the size of the strange quark. Since the size of the ordinary quark is already known from the pion radius, the kaon radius provides us with the size of the strange quark. Typically we have $\gamma_u = 1.225 GeV^{-1}$ and $\gamma_s = 0.200 GeV^{-1}$. We don't have any experimental data concerning charmed and bottom quarks; we choose values that are smoothly drecreasing with size, namely $\gamma_c = 0.04 GeV^{-1}$ and $\gamma_b = 0.013 GeV^{-1}$.

Concerning the parameters appearing in the strong potential, we decided to let them unchanged, except the masses of the u and d quarks that are now split. In order to minimize the number of free parameters, we also maintain their average value; thus we have at our disposal only one free parameter, the mass difference $\Delta = m_d - m_u$, to try to explain the totality of all known splittings. This parameter is determined on a precise and sensitive value, namely the mass difference for the sigma multiplet.

3.2 Experimental sample

For experimental sample, we consider 10 splittings in the mesonic sector, and 16 splittings in the baryonic sector. They belong to the light and heavy quark sectors. They are typically of order of few MeV. The hierarchy of most of the multiplets can be explained naively just by supposing $m_d > m_u$. Nevertheless, it remains some puzzling questions that cannot be interpreted in this naive scheme. I list some of them here.

- $n - p$ is a positive value (this is normal), but $\pi^+ - \pi^0$ is a larger quantity while naively it should be smaller;
- $\pi^+ - \pi^0$ is positive, while $\rho^+ - \rho^0$ is negative;
- One multiplet in the D sector, namely the D_2, is much smaller than all others in the same sector;
- Σ_c^{++} is experimentally the highest member of the multiplet while a naive argument would have expected it the smallest one;
- in the Ξ_c sector, there is a multiplet which does not fit with the pattern of the others.

Indeed, the electromagnetic splitting is a small quantity (around 1 MeV or less) that is obtained by a difference of large quantities of order 1 GeV. This means that it is a very sensitive quantity which represents a subbtle balance between various ingredients.

In particular, it is of first importance to perform a very precise numerical treatment both in the mesonic and the baryonic sector. An error of 10^{-3} GeV on the absolute masses, which can be considered as good for such an observable, would lead to very incorrect values concerning the splittings. We feel that an accuracy of order 10^{-5} on the absolute masses should be reached in order to insure reliable conclusions. This is why we take a special care in our numerical treatment.

It is also important to take care of all the terms of the electromagnetic potential, because all of them are equally important. In the same spirit, it is crucial to have a good wave function. Changing the strong potential may affect seriously the wave function, without spoiling too much the spectra, and this may have a dramatic repercussion on the theoretical splittings.

3.3 Some approximations

Usually the dressing of the electromagnetic potential is not considered, or only partly, and often the bare one is taken into account in a perturbative way. Of course,

Table 1. Electromagnetic splittings (in MeV) for mesons obtained with an exact treatment based on wave functions resulting from AL1 potential. The total electromagnetic hamiltonian is considered. For information, the experimental data, extracted from [13], are given in column "Exp"

Splitting	Exp	Theo
$\pi^+ - \pi^0$	4.594 ± 0.001	1.69
$\rho^+ - \rho^0$	-0.5 ± 0.7	0.71
$K^0 - K^+$	3.995 ± 0.034	8.11
$K^{0*} - K^{+*}$	6.7 ± 1.2	1.44
$K_2^0 - K_2^+$	6.8 ± 2.8	-0.76
$D^+ - D^0$	4.78 ± 0.1	2.31
$D^{+*} - D^{0*}$	2.6 ± 1.8	1.04
$D_1^+ - D_1^0$	6.8 ± 5	-2.16
$D_2^+ - D_2^0$	0.1 ± 4	-2.23
$B^0 - B^+$	0.33 ± 0.28	-1.71

changing the free parameters can give correct results, but we think that using a dressed expression is more satisfactory. It allows to have a complete consistency between the treatments for spectra, for charge radii, and for the electromagnetic splittings.

The use of the dressed potential, instead of the bare one is crucial. Indeed, the bare interaction cannot be used in an exact calculation, because it leads to a collapse, due to the presence of the Dirac term. But even in a perturbative treatment, we checked that the differences between bare and dressed potentials can raise 50 MeV, a very important value. We also think that, since the strong potential is already dressed with a gluonic density, it is much more consistent and satisfactory to dress also the electromagnetic potential.

Using a perturbative procedure can give catastrophic results for certain states. In fact the fault is not due to the electromagnetic potential, but mainly to the strong hyperfine term that depends strongly on the mass difference Δ.

Also, all terms of the electromagnetic potential are important, including the contact term that is usually neglected, but it is impossible to say which term is the leading one. Nevertheless, we have remarked that the contact term has a tendancy to spoil the result. We have no explanation for this bad feature.

All these remarks are the consequence that the splittings are a small quantity obtained from large contributions.

3.4 Meson splittings

The results for the splittings in the meson sector are presented in Table 1.

All the terms of the electromagnetic potential are taken into account and an exact numerical treatment is performed. The results are not very good. This is in part due to the fact that the only free parameter has been fitted in the baryonic sector (on the sigma multiplet). We could have obtained much better results, by fitting for example the pion multiplet, but in that case the baryonic sector would have been spoilt a lot. Let us also stress that the results very much depend on the strong potential. Using a different potential gives results appreciably different. In this case, none of the mentioned puzzles can find a solution, except may be the charmed resonances. Once again, we are faced to the problem of having a consistent description of mesons and baryons.

Table 2. Same as Table 1 for baryons. The theoretical uncertainty may affect only the last digit

Splitting	Exp	Theo
$n - p$	1.293	1.15
$\Delta^0 - \Delta^{++}$	2.25 ± 0.68	3.72
$\Delta^+ - \Delta^{++}$	1.2 ± 0.6	1.35
$\Sigma^- - \Sigma^0$	4.81 ± 0.04	4.76
$\Sigma^- - \Sigma^+$	8.08 ± 0.08	8.55
$\Sigma^{-*} - \Sigma^{0*}$	2.0 ± 2.4	2.94
$\Sigma^{-*} - \Sigma^{+*}$	0 ± 4	1.96
$\Xi^- - \Xi^0$	6.48 ± 0.24	7.38
$\Xi^{-*} - \Xi^{0*}$	3.2 ± 0.6	2.66
$\Sigma_c^{++} - \Sigma_c^0$	0.35 ± 0.18	0.37
$\Sigma_c^0 - \Sigma_c^+$	0.9 ± 0.4	0.33
$\Sigma_c^{++*} - \Sigma_c^{0*}$	1.9 ± 1.7	0.19
$\Xi_c^0 - \Xi_c^+$	5.5 ± 1.8	3.42
$\Xi_c^{0\prime} - \Xi_c^{+\prime}$	$\simeq 4.2 \pm 3.5$	0.20
$\Xi_c^{+*} - \Xi_c^{0*}$	$\simeq 2.9 \pm 2.0$	-0.25
$\Xi_c^{0**} - \Xi_c^{+**}$	$\simeq 4.1 \pm 2.5$	3.51

3.5 Baryon splittings

The results for the splittings in the baryonic sector are presented in Table 2, with the same conventions. The results look much more in agreement with data. The order of magnitude is the correct one, and when differences are significant, this always correspond to experimental values affected of a large uncertainty. The order in the multiplet is practically always respected, in particular for the Σ_c multiplet, solving thus one puzzle. Owing to the fact that we have only one free parameter at our disposal, these results can be considered as very encouraging.

Let us mention that the absolute masses of the baryons are also in nice agreement with the data. A small discrepancy is seen for light baryons, but it can be attributed to three-body forces that are not considered here.

4 Conclusions

The electromagnetic splittings are very sensitive observables depending upon a lot of ingredients. In consequence, it is of first importance

- to dress the potentials
- to take into account all influent terms in the potentials
- to perform an exact treatment and not only a perturbative one
- to use numerical algorithm that allow precise calculations for both the two-body and the three-body case

The results also depend significantly on the wave function, and it is important to employ a strong potential that gives a good description of both mesaon and baryon sectors.

We were very cautious in all these respects, and are very confident in our conclusions.

Some puzzles have found a solution, but others are still opened questions, and this means that this subject merits further investigations.

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